1. Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

(a) $\overrightarrow{AB} = (8-3)i + (3-(-7))j$

$\boxed{\overrightarrow{AB} = 5i + 10j}$

(b) $|\overrightarrow{AB}|$ = length of vector $\overrightarrow{AB}$

$= \sqrt{5^2 + 10^2}$

$= \sqrt{25+100}$

$= \sqrt{125}$

$= \sqrt{25 \times 5}$

$\boxed{= 5\sqrt{5}}$

(Total for Question is 4 marks)

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2. Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $-5\mathbf{i} - 2\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$,

(2)

(b) find $|\overrightarrow{AB}|$.

Give your answer as a simplified surd.

(2)

a) $\overrightarrow{AB} = \begin{pmatrix} -5 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} -9 \\ 3 \end{pmatrix}$

$= -9\mathbf{i} + 3\mathbf{j}$

b) $|\overrightarrow{AB}| = \sqrt{(-9)^2 + 3^2}$

$= \sqrt{90}$

$= 3\sqrt{10}$

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3. (i) Two non-zero vectors, **a** and **b**, are such that

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$$

Explain, geometrically, the significance of this statement.

(1)

(ii) Two different vectors, **m** and **n**, are such that $|\mathbf{m}| = 3$ and $|\mathbf{m} - \mathbf{n}| = 6$
The angle between vector **m** and vector **n** is 30°

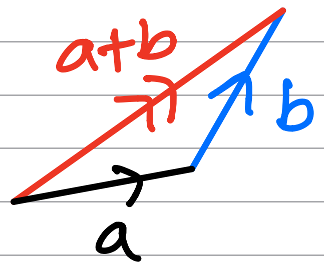
Find the angle between vector **m** and vector **m** – **n**, giving your answer, in degrees, to one decimal place.

(4)

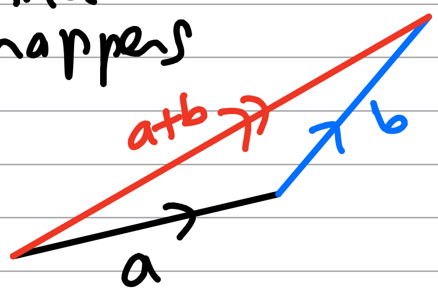
The statement says:

i) The length of the sum of 2 vectors is equal to their individual lengths summed together

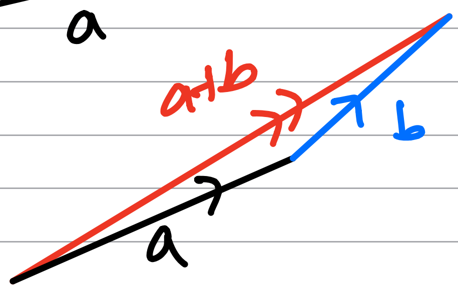
So a and b are vectors that lie on the same straight line (so parallel)



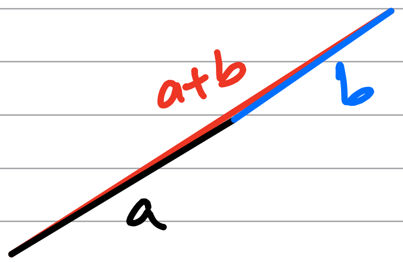
It helps to thinu about what happens as we vary a and b.



here length of a+b is less than (length a) + (length b)



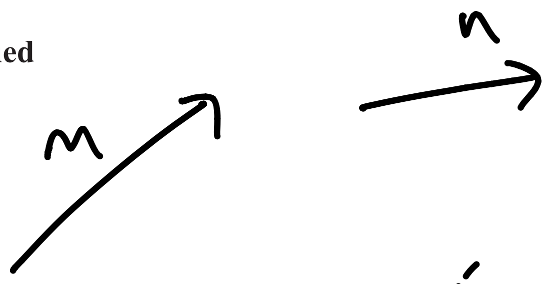
Same here, but |a|+|b| is close to |a+b|.



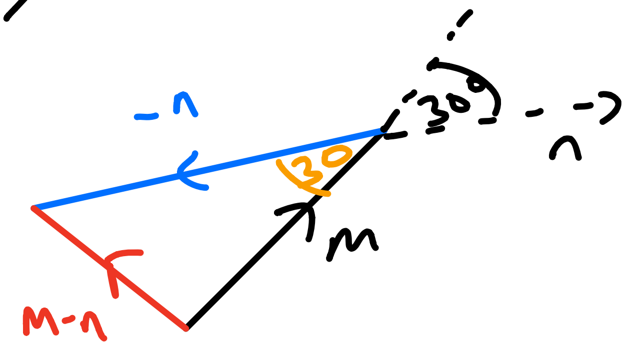
now |a+b| = |a| + |b|
all the sides lie on the same line.

Question continued

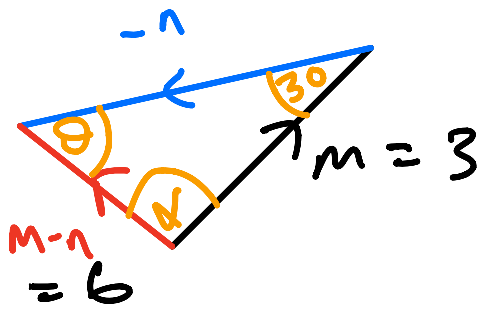
let :



then



$\Rightarrow$



angle required = α

sine rule to find θ :

$$\frac{\sin\theta}{3} = \frac{\sin 30}{6}$$

$$\therefore \theta = 14.5^\circ$$

$$\therefore \alpha = 180 - 30 - 14.5$$

$$= \boxed{135.5^\circ}$$

4. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A coastguard station O monitors the movements of a small boat.

At 10:00 the boat is at the point $(4\mathbf{i} - 2\mathbf{j})$ km relative to O.

At 12:45 the boat is at the point $(-3\mathbf{i} - 5\mathbf{j})$ km relative to O.

The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

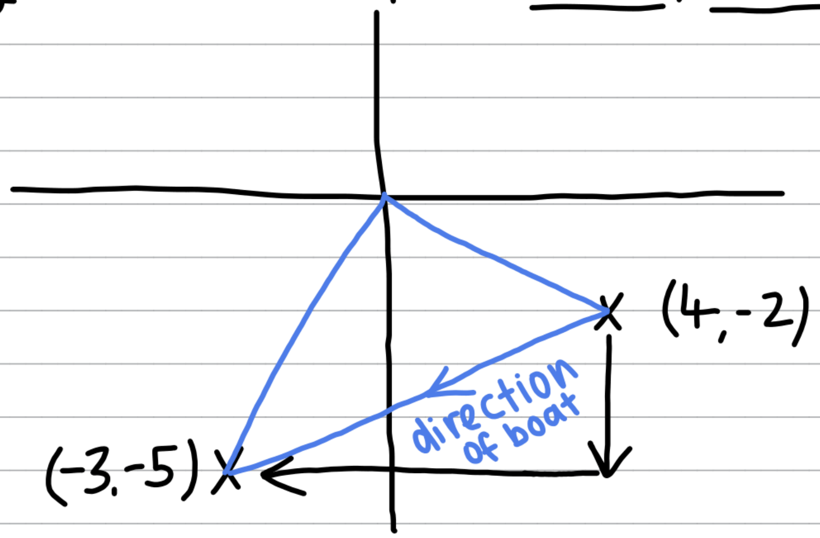
(a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place.

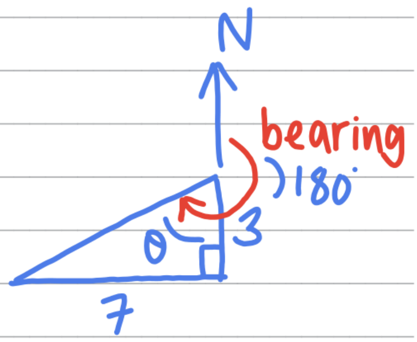
(3)

(b) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)

a) bearing is measured from north, clockwise





direction vector : $\begin{pmatrix}-3\\-5\end{pmatrix} - \begin{pmatrix}4\\-2\end{pmatrix} = \begin{pmatrix}-7\\-3\end{pmatrix}$

angle needed : $\tan\theta = \dfrac{7}{3}$

$= 66.801°$

total bearing $= 180° + 66.8° = \underline{246.8°}$

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Question continued

b) to find speed, need distance travelled

direction vector $\begin{pmatrix}-7\\-3\end{pmatrix}$ so distance $= \sqrt{7^2+3^2} = \sqrt{58}$ km

$\sqrt{(4--3)^2+(-2+5)^2}$

time: 2hrs45 $\Rightarrow$ 2.75 hours

speed $= \dfrac{\sqrt{58}}{2.75} = 2.77\text{ kmh}^{-1}$

(Total for Question is 6 marks)

5. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point O.

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point O.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

(a) prove that the stone passes through O, **(2)**

(b) calculate the speed of the stone. **(3)**

a) $\overrightarrow{OA} = -24i - 10j$

$\overrightarrow{OB} = 12i + 5j$

Now, we want to show that AOB is a straight line. We need to prove that they are parallel

$\overrightarrow{AO} = 24i + 10j$

$\overrightarrow{AO} = 2\overrightarrow{OB}$ 1

$24i + 10j = 2(12i + 5j)$

∴ Since $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$, it is proven that the stone passes through O 1

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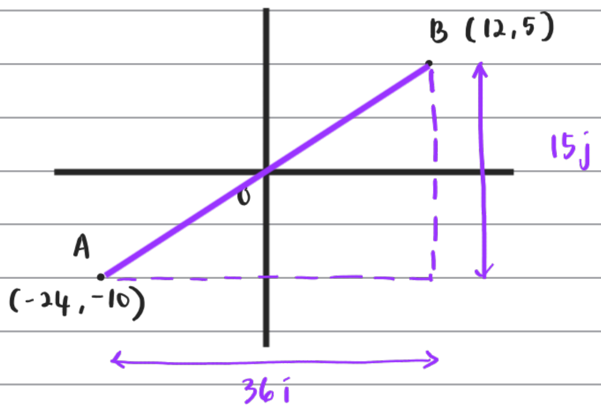
Question continued

b)



$\overrightarrow{AB} = 36i + 15j$

Distance of AB, $|\overrightarrow{AB}| = \sqrt{36^2 + 15^2}$ (1)

$= \sqrt{1521}$

$= 39$ m

speed $= \dfrac{39\text{ m}}{4\text{ s}}$ (1)

$= 9.75\ \text{ms}^{-1}$ (1)

(Total for Question is 5 marks)

6.

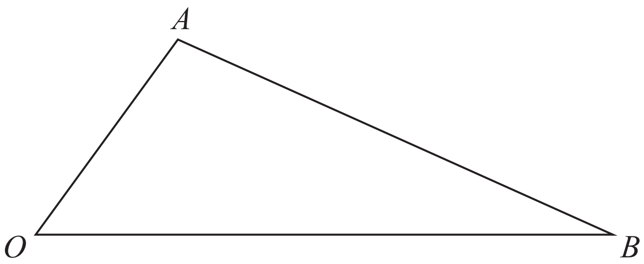


Figure 7

Figure 7 shows a sketch of triangle OAB.

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point M is the midpoint of AB.

The straight line through C and M cuts OB at the point N.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{CM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

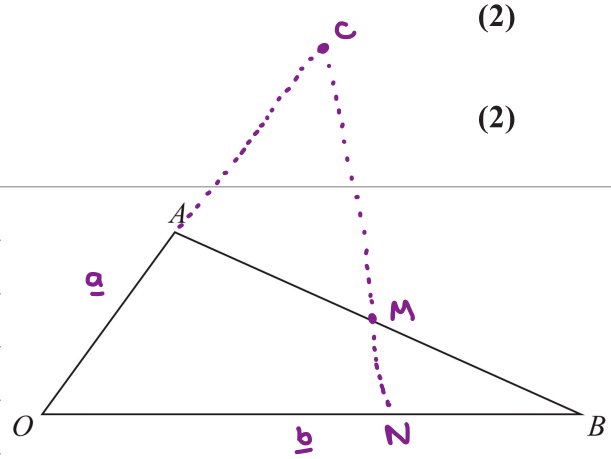
(2)

(b) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that $ON : NB = 2 : 1$

(2)



a) $\overrightarrow{CM} < \underline{a}, \underline{b}$?

$\Rightarrow \overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM}$

$\Rightarrow \overrightarrow{CM} = -\underline{a} + \frac{1}{2}(-\underline{a} + \underline{b})$

$\overrightarrow{CA} = -\overrightarrow{OA} = -\underline{a}$

$\overrightarrow{AM} = \frac{1}{2}(-a + b)$

$\Rightarrow \overrightarrow{CM} = -\underline{a} - \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$ 1

$\Rightarrow \overrightarrow{CM} = -\frac{3}{2}\underline{a} + \frac{1}{2}\underline{b}$ 1

$\vec{ON} = \vec{OC} + \vec{CN}$

$\vec{OC} = 2\underline{a}$

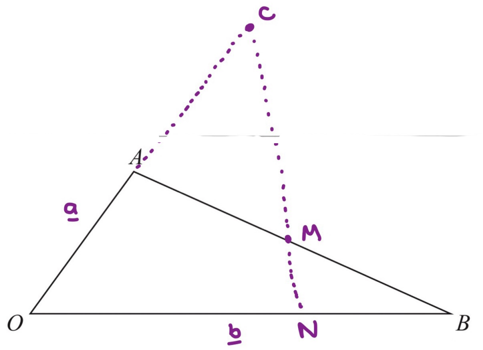
$\vec{CN} = \lambda\vec{CM}$ (1)

$= \lambda\left(-\frac{3}{2}\underline{a} + \frac{1}{2}\underline{b}\right)$

$\vec{ON} = 2\underline{a} + \lambda\left(-\frac{3}{2}\underline{a} + \frac{1}{2}\underline{b}\right)$

$\vec{ON} = 2\underline{a} - \frac{3}{2}\lambda\underline{a} + \frac{1}{2}\lambda\underline{b}$

$\vec{ON} = \left(2 - \frac{3}{2}\lambda\right)\underline{a} + \frac{1}{2}\lambda\underline{b}$ as required. (1)



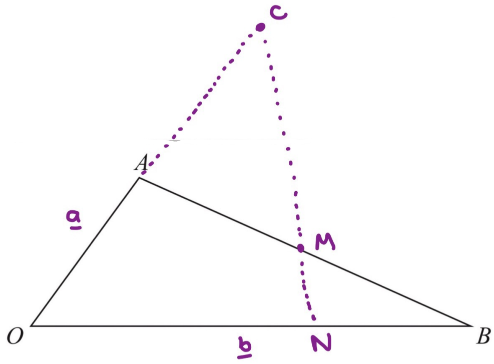
c)

$\vec{NB} = 2\vec{ON}$

$\left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow 4 = 3\lambda \Rightarrow \lambda = \frac{4}{3}$ (1)

$\Rightarrow \vec{ON} = \frac{2}{3}\underline{b} \Rightarrow \vec{NB} = \frac{1}{3}\underline{b}$

$\Rightarrow \vec{ON} : \vec{NB} = \underline{\underline{2:1}}$ as required. (1)



7. Relative to a fixed origin, points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively.

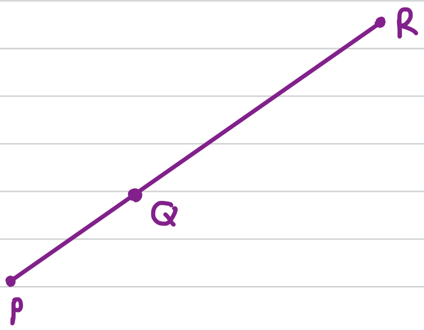
Given that

- P, Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$

(3)



$$\overrightarrow{QR} = \frac{2}{3}\overrightarrow{PR}$$

$$\underline{r} - \underline{q} = \frac{2}{3}(\underline{r} - \underline{p}) \quad 1$$

$$\Rightarrow \underline{r} - \underline{q} = \frac{2}{3}\underline{r} - \frac{2}{3}\underline{p}$$

$$\Rightarrow \underline{q} = \underline{r} - \frac{2}{3}\underline{r} + \frac{2}{3}\underline{p} \quad 1$$

$$\Rightarrow \underline{q} = \frac{1}{3}\underline{r} + \frac{2}{3}\underline{p}$$

$$\Rightarrow \underline{q} = \underline{\frac{1}{3}(\underline{r} + 2\underline{p})} \text{ as required} \quad 1$$